IITB-Monash Research Academy

An Indian-Australian research partnership

Project Title:	A general framework for a posteriori error estimators and adaptive mesh refinement	
Project Number	**IMURA0439**	
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Research Academy Themes:

Highlight which of the Academy's Theme(s) this project will address?
(*Feel free to nominate more than one. For more information, see www.iitbmonash.org)*

1. Advanced computational engineering, simulation and manufacture
2. Infrastructure Engineering
3. Clean Energy
4. Water
5. Nanotechnology
6. Biotechnology and Stem Cell Research

The research problem

Diffusion Partial Differential Equations permeate numerous physical models, among which some that display features of very different scales. This is for example the case in reservoir engineering (oil recovery, carbon storage...) in which the interface/faults between geological layers, or the well bores, have different scales from the scale of the reservoir. A brute application of classical numerical methods to capture these features of different scales requires to use very thin meshes, which leads to unreasonable computational costs. Mesh refinement is therefore the preferred method; this method consists in starting from coarse meshes and, instead of refining them in the whole domain, in finding appropriate indicators to locate the features of low dimension and to refine the mesh only around these features. This mesh refinement has been studied for any numerical methods, but new methods are regularly proposed to tackle some specific issues in the approximation of the models, and the study of mesh refinement for those is still to be done.

Instead of looking at each particular new method, we propose in this project to find common features between all (or most of) the numerical schemes developed for diffusion equations, and to use these features to design *a posteriori* estimators that would be applicable to a wide range of schemes.

The research will start from the framework of Gradient Schemes, which has been developed to provide unified convergence analyses for many different numerical methods (Finite Elements, Mixed Finite Elements, Finite Volumes, Galerkin approximations, etc.) and many different diffusion models (linear as well as non-linear). This framework identifies 4 properties, shared by all these schemes are that are sufficient (and crucial) for the convergence analyses. We will try to identify, from these properties or more specific ones, local indicators for mesh refinement. We will both analyse the theoretical quality of these indicators, and their practical efficiency on particular benchmarks.

Project aims

Define the aims of the project

We aim to

(a) Design generic *a posteriori* estimators for a wide family of numerical methods for diffusion equations
(b) Analyse the quality of these estimators: optimality, convergence.
(c) Implement these estimators for specific schemes and compare their quality with existing, scheme-specific, estimators.

Expected outcomes

(a) Scheme-independent *a posteriori* mesh refinement indicators that can be applied to many different numerical methods (including methods not yet developed but satisfying general assumptions).
(b) Numerical implementation of these estimators for some particular methods.
(c) Quality international research publications;
(d) Joint supervision of a Ph.D. student from India which will help to boost the research in Numerical Analysis in the country;
(e) Initiating collaborative research work between Monash University and IIT Bombay in the broad area of numerical analysis and scientific computing.

How will the project address the Goals of the above Themes?

Advanced computational engineering, simulation and manufacture

We would be addressing the first and second components in the above mentioned theme. Computational PDEs is an extremely active topic internationally. The proposed project aims at providing tangible progress in the direction of the growth of the topic.

Capabilities and Degrees Required

Candidates should

1. have a strong mathematical background in post graduation;
2. have done courses in Partial differential Equations & Functional Analysis in Masters level,
3. Knowledge and aptitude in computer languages – like Matlab, Fortran90, C or C++ – is essential.